Week 1 Study Summary: Asymptotic Analysis and Big-O Cheat Sheet

Z5007 Programming and Data Structures · Oct 5–9, 2026 · Dr. Innocent Nyalala

1. The RAM Cost Model and Asymptotic Notation

Under the Random-Access Machine (RAM) model, basic arithmetic, array indexing, and branch instructions each cost one unit step. Asymptotic classes drop constant factors and lower-order terms for large n:

- **Upper Bound ($f(n) \in O(g(n))$):** $\exists\, c > 0, n_0 \geq 1$ such that $0 \leq f(n) \leq c\, g(n)$ for all $n \geq n_0$.
- **Lower Bound ($f(n) \in \Omega(g(n))$):** $\exists\, c > 0, n_0 \geq 1$ such that $f(n) \geq c\, g(n)$ for all $n \geq n_0$.
- **Tight Bound ($f(n) \in \Theta(g(n))$):** Both $O(g(n))$ and $\Omega(g(n))$ hold simultaneously.

2. Complexity Hierarchy and Growth Rates

Class	Name	Representative Operation	Max n in 1 ms (10^6 ops)
$O(1)$	Constant	Array index, `set` lookup	Any n
$O(\log n)$	Logarithmic	Binary search, AVL height	$> 10^{100}$
$O(n)$	Linear	Single-pass scan, min/max	10^6
$O(n \log n)$	Linearithmic	Merge sort, heap sort	$\approx 50,000$
$O(n^2)$	Quadratic	Pairwise inversion scan	$1,000$
$O(2^n)$	Exponential	Brute-force subset scan	≈ 20

Empirical Log-Log Verification

For any power-law algorithm $T(n) \approx c\, n^b$, fitting a straight line to $(\log n, \log T(n))$ recovers slope b. In Lab 1, pairwise inversion counting gives $b \approx 2.0$, whereas Python `set` membership remains flat ($b \approx 0$) relative to $O(n)$ list scanning.