Week 16 Summary: P, NP, Polynomial Reductions, and Approximation

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1. Complexity Classes P, NP, and NP-Completeness

Class **P** contains decision problems solvable in deterministic polynomial time; **NP** contains problems whose "yes" certificates verify in polynomial time. Problem B is **NP**-hard if every $A \in \mathbf{NP}$ reduces to B in polynomial time ($A \leq_P B$), and **NP**-complete if $B \in \mathbf{NP}$ as well.

2. Classic Reduction Chain

$$\text{Circuit-SAT} \leq_P \text{3-SAT} \leq_P \text{Independent Set} \leq_P \text{Vertex Cover}.$$

From m 3-CNF clauses, build m literal triangles plus edges linking complementary literals $(x, \neg x)$ across clauses: a satisfying assignment exists if and only if an independent set of size m exists.

3. Approximation Algorithms

When exact polynomial solvers are unlikely, greedy maximal matching yields a 2-approximation for Minimum Vertex Cover ($|C| \leq 2\,\mathrm{OPT}$), and MST preorder doubling yields a 2-approximation for Metric TSP.